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Quark–gluon plasma as a strongly coupled color-Coulombic plasma

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Abstract. We show that the extensively studied equation of state (EOS) of strongly coupled QED plasma fits the recent lattice EOS data of gluon plasma remarkably well, with appropriate modifications to take account of color degrees of freedom and the running coupling constant. Hence we conclude that the quark–gluon plasma near the critical temperature is a strongly coupled color-Coulombic plasma.

Quark–gluon plasma (QGP), the deconfined state of quarks and gluons, is a prediction of quantum chromodynamics (QCD). In search of QGP, experiments are on at CERN, BNL etc., where heavy ions are accelerated to relativistic energies and made to collide. At sufficiently large energy (>few GeV), the fireball formed by a collision may be a QGP which then expands, cools, and freezes out into hadrons. These hadrons, photons, leptons etc. are detected and analyzed to see whether a QGP is formed or not. The expansion of QGP, generally described by relativistic hydrodynamics, affects the experimental observations. In the description of hydrodynamics, the EOS of QGP is needed to complete the set of fluid equations. At present most of the analyses of experimental results are based on an ideal equation of state, where quarks and gluons are non-interacting and pressure or energy density are proportional to the forth power of the temperature. In the theoretical calculations of various signatures of QGP based on the hydrodynamic expansion, it is assumed that QGP is an ideal plasma.

However, recent lattice-simulation results show that gluon plasma is not ideal and is non-ideal even up to five times the critical temperature (T_c) [1]. A lot of attempts are made [2–4] to explain this numerical experiment based on the properties of QCD, confinement and asymptotic freedom, the properties of plasma, etc. All of them are, so far, not able to explain the data satisfactorily.

In [2], it was tried to fit the lattice results assuming Coulombic interaction with a running coupling constant, confinement in the form of a bag constant, and a low momentum cut-off in the evaluation of the partition function. In [3], gluons are assumed to be quasi-particles with a mass proportional to the plasma frequency without any confinement. Here also a modified, temperature-dependent, running coupling constant is used to fit the old lattice results of [5]. As we will see, it does not fit the recent more-refined data [1]. Earlier we [4] had assumed a Cornel potential interaction [6], Coulomb plus confinement, between a quark and an antiquark, and used Mayer's cluster expansion method to derive the EOS. This EOS was used to fit lattice results on a gluon plasma with partial success. Especially near the critical temperature the fit was not good. We suspected that it might be due to the fact that our theory was for a weakly interacting system and hence valid only in the high temperature limit. To explain the lattice results near T_c, we need a theory of a strongly interacting system. In other words, near T_c QGP may be a strongly coupled plasma. This is exactly what we find in this paper, namely that QGP is a strongly coupled color-Coulombic plasma.

Generally, when we say plasma, it is a quasi-neutral system of charged particles which exhibits collective effects. The so-called plasma parameter Γ, which is the ratio of the average potential energy to the average kinetic energy of particles, is assumed to be weak (<<1). A lot of studies on plasma, such as on plasma waves, instabilities and other collective effects, are in this range of Γ. For Γ close to 1 and above, it is a strongly coupled plasma (SCP) which modifies various plasma properties [7]. We see that QGP is also a strongly coupled plasma near T_c. Partially analytic and partially numerical calculations of SCP do exist in the literature [7]. In particular, the EOS of SCP is extensively studied and parameterized as a function of Γ.

Let us now discuss QGP. We take QGP to be a deconfined, quasi-color-neutral system of quarks, antiquarks and gluons with color-Coulombic mutual interactions. It is similar to a QED plasma except for a few modifications due to color degrees of freedom. It is characterized by the plasma parameter

$$\Gamma \equiv \frac{\langle \mathrm{PE} \rangle}{\langle \mathrm{KE} \rangle} = \frac{\frac{4}{3}\frac{\alpha_s}{r_{av}}}{T}, \tag{1}$$

where we have taken the well known Coulombic interactions used in hadron spectroscopy [6]. Here we have neglected the confinement interactions which may, probably, play a role near the phase transition temperature $T_{\rm c}$. For the typical values of $\alpha_{\rm s} \approx 0.5$, $r_{\rm av} \approx 1\,{\rm fm}$ and near the critical temperature $T_{\rm c} \approx 200\,{\rm MeV}$, we estimate Γ to be $\approx 2/3$. Hence QGP is a strongly coupled plasma. Compared to QED, the fine structure constant α is replaced by $4\alpha_{\rm s}/3$ in the Coulomb interaction term. $r_{\rm av}$ may be estimated as $r_{\rm av} = (3/4\pi n)^{1/3}$ and hence

$$\Gamma = \left(\frac{4\pi n}{3}\right)^{1/3} \frac{4}{3}\frac{\alpha_{\rm s}}{T}, \tag{2}$$

where n is the number density. As we discussed earlier, there exists an EOS for a strongly coupled Coulombic system as a function of Γ. It is a non-relativistic and classical result, including the definition of Γ, whereas QGP is a relativistic and quantum system. Still, since the Coulomb interaction is common both in QGP and SCP, it is interesting to see how far the existing EOS of SCP, modified to QGP, would be able to explain the non-ideal behavior reported in lattice calculations. Hence we modify the QED non-relativistic energy density, $\varepsilon_{\rm QED} = (3/2 + u_{\rm ex}(\Gamma))nT$, for strongly coupled relativistic QGP (SCQGP) to

$$\varepsilon = (3 + u_{\rm ex}(\Gamma))nT, \tag{3}$$

where the first term, $3nT$, corresponds to the ideal EOS which, in our case, may be written as $\varepsilon_{\rm s} \equiv 3a_{\rm f}T^4$. $a_{\rm f}$ is a constant which depends on degrees of freedom. The non-ideal contribution to the EOS, $u_{\rm ex}(\Gamma)$, is given by

$$u_{\rm ex}(\Gamma) = \frac{u_{\rm ex}^{\rm Abe}(\Gamma) + 3\times 10^3\Gamma^{5.7}u_{\rm ex}^{\rm OCP}(\Gamma)}{1 + 3\times 10^3\Gamma^{5.7}}, \tag{4}$$

where we have taken the same $u_{\rm ex}^{\rm Abe}$ and $u_{\rm ex}^{\rm OCP}$ as used in SCP. They are given by

$$u_{\rm ex}^{\rm Abe}(\Gamma) = -\frac{\sqrt{3}}{2}\Gamma^{3/2} - 3\Gamma^3\left[\frac{3}{8}\ln(3\Gamma) + \frac{\gamma}{2} - \frac{1}{3}\right], \tag{5}$$

and

$$\begin{aligned} u_{\rm ex}^{\rm OCP} &= -0.898004\Gamma + 0.96786\Gamma^{1/4} \\ &\quad + 0.220703\Gamma^{-1/4} - 0.86097. \end{aligned} \tag{6}$$

$u_{\rm ex}^{\rm Abe}$ was derived by Abe [8] exactly in the giant cluster-expansion theory and is valid for $\Gamma < 0.1$. $\gamma = 0.57721\cdots$ is Euler's constant. $u_{\rm ex}^{\rm OCP}$ was evaluated by a computer simulation of a one-component plasma (OCP), where a single species of charged particles is embedded in a uniform background of neutralizing charges, and it is valid for $1 \le \Gamma < 180$. $u_{\rm ex}(\Gamma)$ is valid for all $\Gamma < 180$. $u_{\rm ex}(\Gamma)$ is derived for a strongly coupled Coulombic plasma, and is rigorously verified. It may be valid for any Coulombic plasma with an appropriate change in the charge or the coupling constant α, e.g., in the case of dusty plasma [9] $\alpha \to Z^2\alpha$, where Z is the number of charges on the dust particle. Similarly, in our case, for color-Coulombic plasma we replace α by $\alpha_{\rm s}$ and use this along with appropriate color factors. Finally, we get

$$e(\Gamma) \equiv \frac{\varepsilon}{\varepsilon_{\rm s}} = 1 + \frac{1}{3}u_{\rm ex}(\Gamma). \tag{7}$$

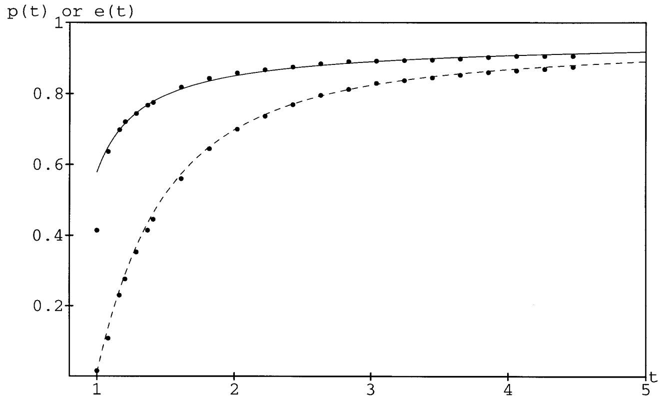

Fig. 1. Plots of $p(t) \equiv P/P_{\rm s}$ (dashed curve) and $e(t) \equiv \varepsilon/\varepsilon_{\rm s}$ (continuous curve) as a function of $t \equiv T/T_{\rm c}$ from our model and lattice results (dots), respectively

Using the relation $\varepsilon = T(\partial P/\partial T) - P$, we get the pressure

$$p(T) \equiv \frac{P}{P_{\rm s}} = \left(\frac{P_{\rm c}}{a_{\rm f}T_{\rm c}} + 3\int_{T_{\rm c}}^{T} {\rm d}\tau\,\tau^2 e(\tau)\right)/T^3, \tag{8}$$

where $P_{\rm s} \equiv a_{\rm f}T^4$ and $P_{\rm c}$ is the pressure at critical temperature $T_{\rm c}$.

Let us consider a gluon plasma which is widely studied in lattice simulations of QCD. Taking $n \approx 1.11a_{\rm f}T^3$, from (2), we get $\Gamma = (4\pi a_{\rm f}/3)^{1/3} g_{\rm c}\alpha_{\rm s}$. Here $g_{\rm c}$ is a parameter which will take account of uncertainties in the definition of Γ, n, and the type of plasma. $g_{\rm c}$ may be different for QED plasma, quark–antiquark plasma, gluon plasma etc. The temperature dependence of $e(\Gamma)$ seen in lattice data can only come from $\alpha_{\rm s}$ in our model by using the running coupling constant $\alpha_{\rm s}(T)$. This is not new, and in earlier work on the EOS of gluon plasma [2,3], a crucial temperature dependence comes from $\alpha_{\rm s}(T)$. Let us use the two-loop order running coupling constant $\alpha_{\rm s}(T)$:

$$\alpha_{\rm s}(t) = \frac{2\pi}{11\ln(t/t_0)}\left(1 - \frac{51}{121}\frac{\ln(2\ln(t/t_0))}{\ln(t/t_0)}\right), \tag{9}$$

where $t \equiv T/T_{\rm c}$ and $t_0 \equiv \Lambda/T_{\rm c}$. Λ is the QCD scale parameter. Thus we have two parameters t_0 and $g_{\rm c}$ in the problem, adjusted to get the best fit to lattice data [1]. $e(t)$ is given by (7) and $p(t)$, from (8), is given by

$$p(t) \equiv \frac{P}{P_{\rm s}} = \left(p_{\rm c} + 3\int_1^t {\rm d}\tau\,\tau^2 e(\tau)\right)/t^3. \tag{10}$$

In Fig. 1 we plotted $e(t)$ and $p(t)$, and we obtained a remarkable good fit to lattice data [1] for $t_0 = 0.5$ and $g_{\rm c} = 1.5$. We have used $a_{\rm f} = 16\pi^2/90$, which is for gluons. The one-loop order temperature-dependent running coupling constant was used earlier in [2] to fit the lattice EOS of gluons with $t_0 \equiv \Lambda/T_{\rm c}$. We find that the one-loop order $\alpha_{\rm s}(T)$ in our model is not sufficient to get a good fit to the lattice data and we need a two-loop order $\alpha_{\rm s}(T)$. Note that it is consistent with lattice calculations where, in fact, a two-loop renormalized $\alpha_{\rm s}(T)$ was used. In Fig. 2, we plotted $\alpha_{\rm s}(t)$ and $\Gamma(t)$ as a function of t. We see that, even

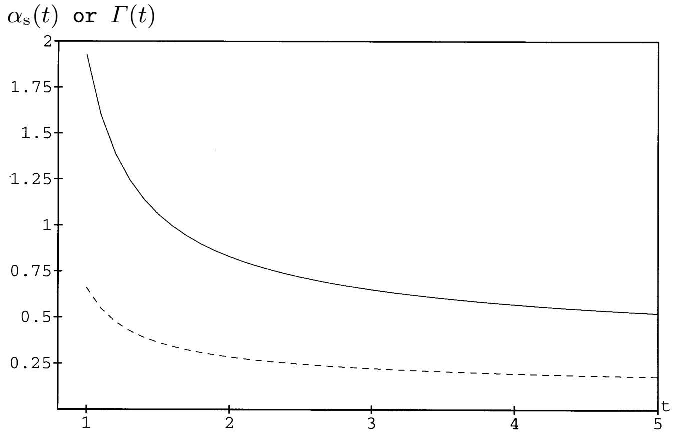

Fig. 2. Plots of $\alpha_s(t)$ (dashed curve) and $\Gamma(t)$ (continuous curve) as a function of t

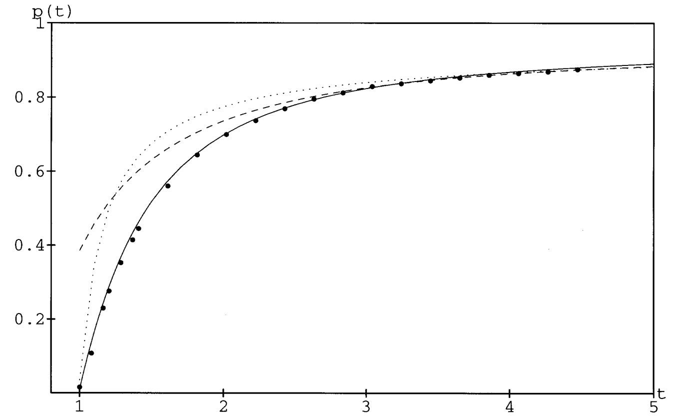

Fig. 3. Plots of the best fits for $p(t)$ from three models, [3] (dotted curve), [4] (dashed curve) and the present model (continuous curve), and lattice results (dots) as a function of t

at $T = 5T_c$, gluon plasma is a strongly coupled plasma, i.e., Γ is of the order of 1. The running coupling constant is also not small. Since $\Gamma \approx 0.5$ at $T = 5T_c$, from our earlier discussion on internal energy (4), the main contribution to $u_{ex}(\Gamma)$ comes from $u_{ex}^{OCP}(\Gamma)$. This is also exactly what we see in our calculations. The contribution from the u_{ex}^{Abe} term is negligibly small. We also find that the dominant term in u_{ex}^{OCP} (6) is the second term. However, the first term, which looks similar to the one-gluon exchange term of perturbative QCD, dominates over the other three terms taken together. Even beyond $T = 5T_c$, we find that Γ decreases very slowly and approaches the ideal gas limit only asymptotically. For very large T or $\Gamma < 0.1$, u_{ex}^{Abe} dominates over u_{ex}^{OCP}. Hence, if QGP is formed in relativistic heavy-ion collisions with a temperature of a few hundred MeV, it may be a strongly coupled quark–gluon plasma, as discussed here, and one needs to analyze the expansion of the fireball using a non-ideal EOS of SCQGP.

It is interesting to compare a few earlier theories with our present theory. In Fig. 3, the best fits for $p(t)$ for three models ([3], [4] and the present model) are plotted. As we see, the fit of lattice data with theory improves as we go from quasi-particle theory and our earlier result to the present result.

In conclusion, we obtained a remarkably good fit to the recent lattice result [1] using the EOS of strongly coupled plasma of QED [7] with a modification for color degrees of freedom and with a running coupling constant. Hence we conclude that QGP is a strongly coupled Coulombic plasma of color charges. Of course, we have two parameters g_c and t_0 in our model. However, t_0 is related to the QCD scale parameter Λ and to the critical temperature T_c, and hence it may be used to predict T_c if we know Λ. g_c is an unknown parameter and may be derived only by a more fundamental theoretical calculation of the EOS for SCQGP. However, here g_c is 1.5, and it is of the same order as the Casimir operator (C_f) for gluons, which is 3. Hence it is not completely unreasonable. It should be noted that in our model we have not included confinement effects, such as glueballs etc., which may be important around the phase transition temperature T_c. Our result is not good close to T_c probably because of this. Just like in SCP, SCQGP may have dramatic effects on various signatures, collective effects and other properties. They need to be recalculated before we confirm the existence of QGP in relativistic heavy-ion collisions. E.g., the screening length decreases rapidly near the critical temperature and we may have serious consequences in J/ψ suppression results.

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